a-g Integrated Math 2

Connecting Waters Charter School (053706)

Basic Course Information

Abbreviations:	Course code:
a-g Integrated Math 2A	5E0042
a-g Integrated Math 2B	5E0043

Length of course:
Full Year (2 semesters; 3 trimesters; 4 quarters)

Subject area: Mathematics (“c”)
Discipline: Mathematics II

UC honors designation: No

Grade levels: 9th, 10th, 11th, 12th

Course learning environment: Classroom

Is this course an integrated course? No

Course Description

Overview:
Integrated Math 2 is the second course of a three year integrated high school sequence that combines topics in Geometry, Algebra, and Statistics and Probability. The study of geometry will build upon the students' knowledge of dilations and proportional reasoning to include a formal understanding of similarity, and use similarity to solve problems, including those involving right triangle trigonometry, and special right triangles. Student will learn to develop formal proofs involving various methods and a variety of geometric theorems. Study of geometry will also include extensive study of circles and volume formulas and use them to solve problems. In this course, students will build upon the algebra and function concepts that were introduced in Integrated Math 1, by extending the study of linear and exponential functions to include quadratic and other polynomial functions. They will use the appropriate functions in modeling problems. Students will also extend their understanding of quadratic functions to include complex numbers so that all quadratics can be solved. Students continue to build on statistics and probability studied in the middle school grades to include conditional probability, and probability of compound events.

Prerequisites: Integrated Math 1 or Algebra 1, required

Corequisites: None

Course content:

Reasoning and Proof
The unit begins with basic constructions introducing students to the tools of geometry as well as

points, segments, rays, angles and lines. Students are then introduced to inductive and deductive reasoning. They examine patterns in number sequences and in sequences of geometric figures. They learn to make a conjecture and prove it false by finding a counterexample or prove it to be true using deductive reasoning in a mathematical proof.

Students will begin proofs with algebraic equations and then prove geometric relationships using given information, definitions, properties, postulates, and theorems. Students will learn the logic of conditional statements and how they are expressed in definitions, properties, postulates and theorems.

Sample Assignment: Students will produce a portfolio of basic compass and straightedge constructions: a segment congruent to a given segment, an angles congruent to a given angle, perpendicular bisector of a segment, and an angle bisector. They will check the accuracy of their constructions using protractors and rulers, and answer the following questions:

- What are the two tools used to make constructions?
- How can a compass be used as a measuring tool?
- Describe the difference in accuracy between sketching a figure, drawing a figure with a ruler and protractor, and constructing a figure.

Students learn the meaning of bisecting both a segment and an angle. They learn the meaning of perpendicular, and they become familiar with the tools of geometry and the limitations of the accuracy of each tool.

Proving Theorems about Lines and Angles
This unit expands on the students' knowledge of lines and angles and proofwriting. Students explore the the idea that not all lines and not all planes intersect. When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs. The special angle pairs formed by parallel lines and a transversal are either congruent or supplementary.

Students will prove and use theorems about certain pairs of angles that can be used to decide whether two lines are parallel or perpendicular. Students will use the parallel postulate to prove the Triangle AngleSum Theorem. Students will expand their construction skills by learning to construct parallel and perpendicular lines.

Sample Assignment: Students use a straightedge to draw an triangle and label the interior angles 1, 2, and 3 on stiff paper. They then cut out the triangle and trace it onto blank paper, using the same angle labels. Then extend the sides of the triangle to form an exterior angle at each vertex, labeling them 1e, 2e, and 3e respectively. Students the tear the corners off of the triangle and arrange the

angles on the drawing on the paper, putting vertices and sides together. Students should make a conjecture about the sum of the three angles of a triangle as well as the relationship between the measure of an exterior angle and the measures of the interior angles.

Students learn that the sum of the three angles of a triangle is 180 degrees. They will then prove this conjecture using the Parallel Postulate and theorems about parallel lines cut by a transversal that were learned previously in the unit. They will also conjecture that the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Congruent Triangles
This unit builds upon the students' understanding and skills related to angles and triangles.

Students will first learn that two figures are congruent if all of the corresponding parts are congruent. Students will then learn that two triangles can be proven congruent without proving all of the corresponding parts congruent. Specifically, they are congruent if three pairs of corresponding sides (SSS) are congruent, two pairs of corresponding sides and one pair of corresponding angles (SAS or HL) are congruent, or one pair of corresponding sides and two pairs of corresponding angles (ASA or AAS) are congruent. Students combine visual and deductive skills to determine triangle congruence, determining which postulate or theorem to use to prove triangle congruence, and learn to analyze diagrams that may involve overlapping triangles. Students build their deductive arguments to prove consequences of congruent triangles, such as two congruent triangles having congruent third sides or altitudes.

Sample Assignment: "Pieces of Proof"

Students work in groups to arrange the statements and reasons of a twocolumn proof related to triangle congruence. Each group is given an envelope with the diagram, Given and Prove statements, as well as strips of paper that are either statements or reasons or the proof.

Students are given the following directions:

1) Determine which strips are statements, and which are reasons

2) Determine which statements are "given" to start constructing the proof

3) Work together in a group to determine a logical argument for the proof. Each student then copies the complete proof, including the diagram.

In this activity, students identify the properties that exist in a given figure and apply postulates and theorems to build a formal proof involving triangle congruency.

Probability
Students expand on their knowledge of probability to measure the likelihood that an event will occur. They will learn counting techniques to find all of the possible ways to complete different tasks or choose items from a list. Probability of compound events can be found by using the probability of

each part of a compound event. They will learn how to use tables to organize data by frequency and to find probabilities. Tables, tree diagrams, and formulas are also used to find conditional probabilities. After studying basic probability concepts, students analyze how the concepts apply in realworld contexts.

Sample Assignment: Working in small groups, each group decides on two different categories that their classmates would fall into based on a single characteristic. Sample categories include plays/does not play sports, has/does not have a parttime job, plays/does not play a musical instrument. Groups use Males and Females as the other category headings, then gather data and complete a twoway frequency table and a probability distribution for their data. They will then use the tables to calculate simple probabilities and compound probabilities.

By gathering their own data and constructing their own twoway frequency table, students will learn and understand the meaning of each entry in a twoway frequency table. They will also learn to use the table to calculate simple and compound probabilities.

Exponents, Polynomials, and Quadratic Functions
This unit will begin with a review of the properties of exponents to simplify products or quotients of powers with the same base, or powers raised to a power, or products or quotients raised to a power. The concept of the zero exponent and negative exponent will also be reviewed. These concepts will then be extended and students will learn that rational exponents can be used to represent radical expressions. The unit continues with an indepth study of polynomial expressions. Students learn to add, subtract, multiply and factor polynomial expressions.

Finally, students learn to solve quadratic equations using a variety of methods. Students graph quadratic functions on the coordinate plane and use the discriminant of the quadratic equation to analyze the number of times the graph crosses the xaxis. They will solve quadratic equations by graphing, factoring, completing the square, and using the quadratic formula. Complex numbers will be introduced and then used to solve to find all solutions to a quadratic equation, including those with imaginary solutions. Student will use quadratic functions that represent realworld situations and determine if linear, quadratic, or exponential functions appropriately model a set of data.

Sample Assignment: Given a set of data, students will perform the following tasks to determine which modellinear, quadratic, or exponentialwould be the most appropriate model.

- Make a scatterplot and then analyze the shape of the graph.
- Analyze the first and second differences of the yvalues, as well as ratios of yvalues.
- Use the regression feature of a graphing calculator to find an equation of each model. Then graph each equation with the scatterplot.

Students learn to model realworld data and compare the various modelslinear, quadratic, and exponential.

Right Triangles and Trigonometry
Students will extend their understanding of similar triangles, focusing on special right triangles and

the trigonometric ratios of sine cosine and tangent. Students will study the Pythagorean Theorem and use it to determine if a triangle is right, acute or obtuse. They will then use the Pythagorean Theorem to derive the ratios of the side lengths of the special right triangles, 30 6090 and 454590. They will use the ratios to find a missing side length or to solve problems involving these special triangles. Students will then learn the definitions of the sine, cosine, and tangent ratios. They will use these definitions to set up equations to find a missing side length when one acute angle and one side length is known. They will also learn to use inverse trigonometric functions to find the angle measures when only the side lengths are known.

Students will extend their understanding of indirect measurement to include the employment of trigonometric ratios and angles of elevation and depression. Lastly, students will derive a formula for calculating the area of a regular polygon and apply the ratios of the special triangles or use trigonometry when some of the needed lengths are not directly given.

Sample Assignment: Indirect Measurement Using Trigonometry

Students will make a simple clinometer with a protractor, string, paper clip and a straw. They will use this tool to measure the angle of elevation to the top of a tall object. They will also use a tape measure to measure their distance to the base of the object and the height of their eye.

They will then set up a trigonometric equation to find the height of the object.

Students will compare this method of indirect measurement with the others that they learned in the previous unit. They will discuss the accuracy of each method and possible measurement errors that can be made in the activity. They will also discuss how indirect measurement can be used in real-world situations.

Similarity

This unit expands on students' understanding of similar figures whose corresponding side lengths are proportional and whose corresponding angles are congruent. Students use ratios and proportions to determine whether two polygons are similar and to find unknown side lengths of similar figures. They will learn to prove triangles similar based on the relationship of two or three pairs of corresponding parts: AngleAngle, SideAngleSide, or SideSideSide. Students will apply this knowledge to problems involving indirect measurement, after first proving two triangles similar in a realworld diagram. Students will also learn that the altitude drawn to the hypotenuse of a right triangle forms three similar right triangles. The relationship between the segments in right triangles can be used for indirect measurement as well. Students explore the properties of similarity transformations, dilations, which either enlarge or reduce a figure according to a scale factor. Finally, compositions of rigid motions and dilations can be used to understand the properties of similarity. Two figures are similar if there is a similarity transformation that maps one to the other.

Sample Assignment: Indirect Measurement

Students will use three different indirect measurement techniques involving similar triangles to solve realworld problems. The first will involve using shadows to find the height of a tall object, such as a

flagpole or building. The second will involve using similar triangles to find the distance across a pond or river. The third will involve using a square corner held up to eye level to measure the height of a tall object using right triangle similarity.

Students will reinforce the concept of triangle similarity and using the fact that similar triangles have proportional corresponding side lengths. They will take accurate measurements of known lengths, draw diagrams, show how the triangles in their diagrams are congruent, and set up proportions to find the unknown lengths.

Proving Theorems About Quadrilaterals

In this unit, students examine the properties of quadrilaterals. They use the Triangle Angle Sum Theorem learned in the previous unit to derive a formula for finding the sum of the measures of the interior angles of any polygon, based on the number of sides. Students will use the properties of parallel and perpendicular lines and diagonals to classify quadrilaterals. They will learn that parallelograms have special properties regarding their sides, angles and diagonals. They will also learn that if a quadrilateral's sides, angles, and diagonals have certain properties, then the quadrilateral is a parallelogram. Students will also learn about the special properties of rhombuses, squares, rectangles and trapezoids. For figures placed on the coordinate plane, students will use the formulas for slope, distance and midpoint to classify figures and prove geometric relationships. Students will learn to assign variables to name coordinates of a figure on the coordinate plane, allowing them to prove relationships for a general case.

Sample Assignment: Students are given the coordinates of three points on the coordinate plane. They use the definition and properties of parallelograms as well as the distance formula and slope to determine all possible coordinates of the fourth vertex.

This activity will reinforce the concept that slope can be used on the coordinate plane to determine if two segments are parallel. Students apply the slope formula and the definition of a parallelogram to find the missing points of the figure in this assignment.

Proving Theorems About Triangles

In this unit students expand on their knowledge of triangles learned in the previous unit. They will use triangle congruency to prove the Perpendicular Bisector Theorem and the Angle Bisector Theorem. They will make conjectures based on an inquiry lesson regarding the midsegments of triangles and then learn to use coordinate geometry to prove this conjecture, known as the Triangle Midsegment Theorem. They will learn about the special points of concurrency of a triangle the three perpendicular bisectors indicate the circumcenter, the angle bisectors indicate the incenter, and the altitudes indicate the orthocenter.

Sample Assignment: Students will draw a triangle with a straightedge and label the vertices. Then they bisect the sides of the triangle and label the midpoints, either by folding or with a

compass construction. Then they connect the midpoints to form an inscribed triangle, measure and list the length of each side of the triangles. Form a conjecture about the relationship between a midsegment of a triangle and its third side.

In this activity students will review finding the midpoint of a segment. They will learn that the segment joining two midpoints is a midsegment. They will also discover that the midsegment is half the length of the third side and is parallel to the third side.

Surface Area and Volume
In this unit, students will extend their knowledge of basic area formulas to derive formulas for surface area of prisms, cylinders, pyramids, and cones. They will then use these formulas to solve problems. Students will then focus on the concept of volume of space figures and either derive or prove formulas, beginning with the volume of a prism and then extending this formula to the volume of cylinders, pyramids, and cones. Lastly, the formulas for surface area and volume of spheres will be explored and used to solve problems.

Sample Assignment: Students will make models of a cube, a square pyramid, a cylinder and a cone from four provided net templates and compare the dimensions and models. Then they will fill their openfaced pyramids with rice and transfer its contents to the cube and fill their open faced cones and transfer the contents to the cylinder. They will make conjectures about the relationship between the volume of a prism and a pyramid and the volume of a cone and a cylinder.

Students will learn that the volume of a pyramid is one third the volume of a prism with the same size base and same height. They will also learn that the volume of a cone is one third the volume of a cylinder with the same size base and height.

Circles and Arcs
In this unit, students will explore concepts related to circles. Students will find the length of part of a circle's circumference by relating it to an angle in the circle. They will also find the area of parts of a circle formed by radii and arcs when the circle's radius is known. Students will use their understanding of congruent triangles to prove statements about tangent lines. Students can determine characteristics of circumscribed figures using characteristics of tangent lines.

Students will then broaden their understanding of special segments in circles to include congruent chords and congruent arcs. They will learn that congruent chords are equidistant from the center of a circle and that a diameter that is perpendicular to a chord bisects the chord and its related arc. Students learn the relationship between a central angle and its intercepted arc as well as the relationship between an inscribed angle and its intercepted arc. Corollaries from this theorem lead to observations about congruent inscribed angles, right angles within circles, and the angles of an inscribed quadrilateral. Lastly, students will learn that there are special relationships between intersecting chords, intersecting secants, or a secant and tangent that intersect.

Sample Assignment: Students will work in groups to prepare and present an explanation of one of the theorems in this unit. This presentation should include a proof of the theorem, a drawn example, and a demonstration of how to solve a problem involving the theorem.

Examples of theorems to consider:

- The measure of an inscribed angle is half the measure of its intercepted arc.
- Two inscribed angles that intercept the same arc are congruent.
- An angle inscribed in a semicircle is a right angle.
- The opposite angles of a quadrilateral inscribed in a circle are supplementary.
- The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

Students will thoroughly learn the theorem that is assigned to their group. They will also practice constructing viable arguments and critiquing the reasoning of others by developing a proof of a theorem together with their group. Students will learn other theorems as they listen to the group presentations, take notes, and solve problems.

Course Materials
Textbooks

Title	Author	Publisher	Edition	Website	Primary
Mathematics II	Randall I. Charles, Basia Hall, Dan Kennedy, Laurie E. Bass, Allan E. Bellman, Sadie Chavis Bragg, William G. Handlin, Art Johnson, Stuart J. Murphy, Grant Wiggins	Pearson Education	2014		Yes

Websites:

Title	**Author(s)/Editor(s)/Compiler(s)**	**Affiliated Institution or Organization**	**URL**
Pearson Success Net		Pearson Education	www.pearsonsuccessnet.com